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FROM: George H. Gates  
OUR REF.: ARC9-00-030US1 (MMC)  
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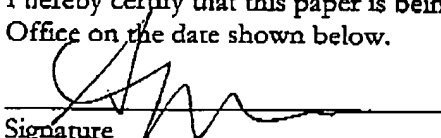
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Group Art Unit:	2123
Title:	METHOD FOR SOLVING STOCHASTIC CONTROL PROBLEMS OF LINEAR SYSTEMS IN HIGH DIMENSION
Our Ref. No.:	ARC9-00-030US1 (MMC)

Please charge all fees to Deposit Account No. 09-0441 of IBM Corporation, the assignee of the present application.

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Confirmation No.: 8338  
Due Date: March 20, 2005

IN THE UNITED STATES PATENT AND TRADEMARK OFFICE

Applicant: Nimrod Megiddo Examiner: Kandasamy Thangavelu  
Serial No.: 09/607,513 Group Art Unit: 2123  
Filed: June 28, 2000 Docket: ARC9-00-030US1 (MMC)  
Title: METHOD FOR SOLVING STOCHASTIC CONTROL PROBLEMS OF LINEAR SYSTEMS  
IN HIGH DIMENSION

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By: [Signature]  
Name: George H. Gates

**MAIL STOP APPEAL BRIEF - PATENTS**

Commissioner for Patents  
P.O. Box 1450  
Alexandria, VA 22313-1450

Dear Sir:

We are transmitting herewith the attached:

- ☒ Transmittal sheet, in duplicate, containing a Certificate of Mailing or Transmission under 37 CFR 1.8.
- ☒ Brief of Appellant(s).
- ☒ Charge the Fee for the Brief of Appellant(s) in the amount of \$500.00 to the Deposit Account.

Please consider this a **PETITION FOR EXTENSION OF TIME** for a sufficient number of months to enter these papers, if appropriate.

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Customer Number 22462

GATES & COOPER LLP

Howard Hughes Center  
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By: [Signature]  
Name: George H. Gates  
Reg. No.: 33,500  
GHG/sjm

Due Date: March 20, 2005

**IN THE UNITED STATES PATENT AND TRADEMARK OFFICE**  
**BEFORE THE BOARD OF PATENT APPEALS AND INTERFERENCES**

In re Application of:	)	
	)	
Inventor: Nimrod Megiddo	)	Examiner: Kandasamy Thangavelu
	)	
Serial #: 09/607,513	)	Group Art Unit: 2123
	)	
Filed: June 28, 2000	)	Appeal No.: _____
	)	
Title: METHOD FOR SOLVING STOCHASTIC	)	
CONTROL PROBLEMS OF LINEAR	)	
<u>SYSTEMS IN HIGH DIMENSION</u>	)	

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**BRIEF OF APPELLANT**

**MAIL STOP APPEAL BRIEF - PATENTS**

Commissioner for Patents  
P.O. Box 1450  
Alexandria, VA 22313-1450

Dear Sir:

In accordance with 37 CFR §41.37, Appellant's attorney hereby submits the Brief of Appellant on appeal from the final rejection in the above-identified application, as set forth in the Office Action dated September 23, 2004.

Please charge the amount of \$500.00 to cover the required fee for filing this Brief of Appellant as set forth under 37 CFR §41.37(a)(2) and 37 CFR §41.20(b)(2) to Deposit Account No. 09-0441 of IBM Corporation, the assignee of the present application. Also, please charge any additional fees or credit any overpayments to Deposit Account No. 09-0441.

**I. REAL PARTY IN INTEREST**

The real party in interest is International Business Machines Corporation, the assignee of the present application.

## II. RELATED APPEALS AND INTERFERENCES

There are no related appeals or interferences for the above-referenced patent application.

## III. STATUS OF CLAIMS

Claims 1-36 are pending in the application.

Claims 1, 3-10, 12, 13, 15-22, 24, 25, 27-34, and 36 were rejected under 35 U.S.C. §103(a) as being unpatentable over Viniotis et al., "Linear programming ... Queueing systems", IEEE, 1988, (Viniotis) in view of Schneider et al. "Stochastic Production scheduling ... demand forecasts", (Schneider).

Claims 2, 14, and 26 were rejected under 35 U.S.C. §103(a) as being unpatentable over Viniotis in view of Schneider and further in view of Dangat et al., U.S. Patent No. 5,971,585 (Dangat).

Claims 11, 23, and 35 were rejected under 35 U.S.C. §103(a) as being unpatentable over Viniotis in view of Schneider and further in view of Hedlund et al., "Optimal control of hybrid systems," IEEE, 1999 (Hedlund).

Claims 1-36 are being appealed.

## IV. STATUS OF AMENDMENTS

No amendments to the claims have been made subsequent to the final Office Action.

## V. SUMMARY OF CLAIMED SUBJECT MATTER

Appellant's invention, as recited in independent claims 1, 13, and 25, is generally directed to an invention for solving, in a computer, stochastic control problems of linear systems in high dimensions. A structured Markov Decision Process (MDP) is modeled in the computer, wherein a state space for the MDP is a polyhedron in a Euclidean space and one or more actions that are feasible in a state of the state space are linearly constrained with respect to the state. One or more approximations are built in the computer from above and from below to a value function for the state using representations that facilitate the computation of approximately optimal actions at any given state by linear programming.

With regard to the claims, Appellants' attorney requests that the Board refer to the specification generally. Specific portions of the specification that directly relate to the claims on appeal include:

(a) at page 3, lines 5-14; at page 4, lines 16-22; at page 7, line 6 through page 21, line 20; and at page 22, line 3 through page 23, line 14, and in FIG. 2 as reference numbers 200-208.

## VI. GROUND S OF REJECTION TO BE REVIEWED ON APPEAL

1. Whether claims 1, 3-10, 12, 13, 15-22, 24, 25, 27-34, and 36 are obvious under 35 U.S.C. §103(a) over Viniotis et al., "Linear programming ... Queueing systems", IEEE, 1988, (Viniotis) in view of Schneider et al. "Stochastic Production scheduling ... demand forecasts", (Schneider).

2. Whether claims 2, 14, and 26 are obvious under 35 U.S.C. §103(a) over Viniotis in view of Schneider and further in view of Dangat et al., U.S. Patent No. 5,971,585 (Dangat).

3. Whether claims 11, 23, and 35 are obvious under 35 U.S.C. §103(a) over Viniotis in view of Schneider and further in view of Hedlund et al., "Optimal control of hybrid systems," IEEE, 1999 (Hedlund).

## VII. ARGUMENT

### A. The Office Action Rejections

In paragraph (4) of the Office Action, claims 1, 3-10, 12, 13, 15-22, 24, 25, 27-34, and 36 were rejected under 35 U.S.C. §103(a) as being unpatentable over Viniotis et al., "Linear programming ... Queueing systems," IEEE, 1998 (Viniotis) in view of Schneider et al., "Stochastic Production scheduling ... demand forecasts," IEEE, 1998 (Schneider). In paragraph (5) of the Office Action, claims 2, 14, and 26 were rejected under 35 U.S.C. §103(a) as being unpatentable over Viniotis in view of Schneider and further in view of Dangat et al., U.S. Patent No. 5,971,585 (Dangat). In paragraph (6) of the Office Action, claims 11, 23, and 35 were rejected under 35 U.S.C. §103(a) as being unpatentable over Viniotis in view of Schneider and further in view of Hedlund et al., "Optimal control of hybrid systems," IEEE, 1999 (Hedlund).

Appellant's attorney respectfully traverses these rejections.

B. The Appellant's Invention

Independent claims 1, 13 and 25 are generally directed to a method for solving, in a computer, stochastic control problems of linear systems in high dimensions. Claim 1 is representative, and comprises:

(a) modeling, in the computer, a structured Markov Decision Process (MDP), wherein a state space for the MDP is a polyhedron in a Euclidean space and one or more actions that are feasible in a state of the state space are linearly constrained with respect to the state; and

(b) building, in the computer, one or more approximations from above and from below to a value function for the state using representations that facilitate the computation of approximately optimal actions at any given state by linear programming.

C. The Viniotis Reference

Viniotis describes linear programming as a technique for optimization of queueing systems. For a significant number of queueing models, that appear in diverse, seemingly unrelated application areas, such as routing, resource allocation and flow control, the optimal policy exhibits a certain "switching-curve" structure. In this paper, we formulate the optimal control problem of such models in a unified way, by using abstract Linear Programming. Using well-known facts from sensitivity analysis of Linear Programs, we show how certain properties of the optimal policy can be easily derived, even in cases where Dynamic Programming (DP) and Stochastic Dominance (SD) arguments fail. A structural property of the optimal value function of the Linear Program, namely piecewise linearity, is exploited to derive properties of the optimal cost function. We also consider additional problems in the realm of queueing system control in which DP or SD approaches are not applicable but Linear Programming may provide useful results.

D. The Schneider Reference

Schneider describes stochastic production scheduling to meet demand. Production scheduling, the problem of sequentially configuring a factory to meet forecasted demands, is a

critical problem throughout the manufacturing industry. The requirements of maintaining product inventories in the face of unpredictable demand and stochastic factory output make the problem difficult. Existing approaches commonly fall into one of two groups: either demand forecasts are discarded and linearizing assumptions are made so methods based on optimal control can be applied, or AI search methods are used to tackle the large search spaces and the ability to handle stochasticity optimally is sacrificed. This paper describes a Markov Decision Process (MDP) formulation of production scheduling which captures stochasticity, while retaining the ability to construct a schedule to meet demand forecasts. The solution to this MDP is a value function, specific to the current demand forecasts, which can be used to generate optimal scheduling decisions online. The paper then describes an industrial application and a reinforcement learning method for generating an approximate value function in this domain. The results demonstrate that in both deterministic and noisy scenarios, value function approximation is an effective technique.

E. The Dangat Reference

Dangat describes a computer implemented decision support tool serves as a solver to generate a best can do (BCD) match between existing assets and demands across multiple manufacturing facilities within boundaries established by manufacturing specifications and process flows and business policies to determine which demands can be met in what time frame by microelectronics (wafer to card) or related (for example disk drives) manufacturing and establishes a set of actions or guidelines for manufacturing to incorporate into their manufacturing execution system to insure the delivery commitments are met in a timely fashion. The BCD tool has six major components, a material resource planning explode or "backwards" component, an optional STARTS evaluator component, an optional due date for receipts evaluator, an optional capacity available versus needed component, an implode "forward" or feasible plan component, and a post processing algorithm.

F. The Hedlund Reference

Hedlund describes optimal control of hybrid systems. This paper presents a method for optimal control of hybrid systems. An inequality of Bellman type is considered and every solution to this inequality gives a lower bound on the optimal value function. A discretization of this "hybrid Bellman inequality" leads to a convex optimization problem in terms of finite-dimensional linear programming. From the solution of the discretized problem, a value function that pre-serves the lower bound property can be constructed. An approximation of the optimal feedback control law is given and tried on some examples.

G. Arguments Directed To The First Grounds for Rejection: Whether claims 1, 3-10, 12, 13, 15-22, 24, 25, 27-34, and 36 are obvious under 35 U.S.C. §103(a) over Viniotis et al., "Linear programming ... Queueing systems", IEEE, 1988. (Viniotis) in view of Schneider et al. "Stochastic Production scheduling ... demand forecasts", (Schneider).

1. Claims 1, 13 and 25

Appellant's claims 1, 13 and 25 are patentable over the references because they recite a novel and nonobvious combination of elements. None of the references, taken individually or in any combination, teaches or suggests this sequence of steps.

Beginning on page 3, the Office Action states the following:

4. Claims 1, 3-10, 12, 13, 15-22, 24, 25, 27-34 and 36 are rejected under 35 U.S.C. 103(a) as being unpatentable over Viniotis et al. (VI) ("Linear programming ... Queueing systems", IEEE, 1988) in view of Schneider et al. (SC) ("Stochastic Production scheduling ... demand forecasts", IEEE, 1998).

4.1 VI teaches Linear programming as a technique for optimization of queueing systems. Specifically, as per Claim 13, VI teaches solving stochastic control problems of linear systems in high dimensions (Page 652, CL1, Para 1; Page 653, CL2, Para 3); comprising:

modeling a structured Markov Decision Process (MDP) (Page 652, CL1, Para 4; Page 652, CL2 Para 6), wherein a state space for the MDP is a polyhedron in a Euclidean space (Page 654, CL2, Lemma 2);

one or more actions that are feasible in a state of the state space are linearly constrained with respect to the state (Page 653, CL1, Para 1 and Para 2; Page 652, CL2, Para 8); and



building a value function for the state using representations that facilitate the computation of approximately optimal actions at any given state by linear programming (Page 653, CL1, Para 9 to Page 654, CL1, Para 4; Page 652, CL2, Para 8).

VI does not expressly teach a computerized apparatus for solving stochastic control problems of linear systems in high dimensions comprising a computer. SC teaches a computerized apparatus for solving stochastic control problems of linear systems in high dimensions comprising a computer (Page 2726, CL1, Para 3 and 4), as that allows the solution of stochastic control problems of linear systems in high dimensions run faster and allows the user to generate the results with varying data (Page 2726, CL1, Para 3). It would have been obvious to one of ordinary skill in the art at the time of Appellant's invention to combine the method of VI with the apparatus of SC that included a computerized apparatus for solving stochastic control problems of linear systems in high dimensions comprising a computer type, as that would allow the solution of stochastic control problems of linear systems in high dimensions run faster and allow the user to generate the results with varying data.

VI does not expressly teach logic performed by the computer, for modeling a structured Markov Decision Process (MDP). SC teaches logic performed by the computer, for modeling a structured Markov Decision Process (MDP) (Page 2726, CL1, Para 3 and 4), as that allows the solution of stochastic control problems of linear systems in high dimensions run faster and allows the user to generate the results with varying data (Page 2726, CL1, Para 3). It would have been obvious to one of ordinary skill in the art at the time of Appellant's invention to combine the method of VI with the apparatus of SC that included logic performed by the computer, for modeling a structured Markov Decision Process (MDP), as that would allow the solution of stochastic control problems of linear systems in high dimensions run faster and allow the user to generate the results with varying data.

VI does not expressly teach logic performed by the computer, for building a value function for the state using representations that facilitate the computation of approximately optimal actions at any given state by linear programming. SC teaches logic performed by the computer, for building a value function for the state using representations that facilitate the computation of approximately optimal actions at any given state by linear programming (Page 2726, CL1, Para 3 and 4), as that allows the solution of stochastic control problems of linear systems in high dimensions run faster and allows the user to generate the results with varying data (Page 2726, CL1, Para 3). It would have been obvious to one of ordinary skill in the art at the time of Appellant's invention to combine the method of VI with the apparatus of SC that included logic performed by the computer, for building a value function for the state using representations that facilitate the computation of approximately optimal actions at any given state by linear programming, as that would allow the solution of stochastic control

problems of linear systems in high dimensions run faster and allow the user to generate the results with varying data.

VI does not expressly teach logic performed by the computer, for building one or more approximations from above and from below to a value function for the state using representations. SC teaches logic performed by the computer, for building one or more approximations from above and from below to a value function for the state using representations (Page 2722, CLI, Para 2; Page 2724, CL2, Para 6), as value function approximation is an effective technique for both deterministic and noisy scenarios (Page 2722, CL1, Para 2); and approximation allows solving large scale MDPs (Page 2722, CL2, Para 2). It would have been obvious to one of ordinary skill in the art at the time of Appellant's invention to combine the method of VI with the apparatus of SC that included logic performed by the computer, for building one or more approximations from above and from below to a value function for the state using representations, as value function approximation would be an effective technique for both deterministic and noisy scenarios and approximation allows solving large scale MDPs.

Moreover, beginning on page 13, the Office Action states the following:

7.1 As per the Appellant's argument that "the Office Action asserts that Viniotis teaches a state space for the MDP is a polyhedron in a Euclidean space, at Page 654, CL2, Lemma 2; however, at the indicated location, Viniotis merely states ... in Viniotis, A is a constraint matrix, not a state space; moreover, Viniotis does not refer to a polyhedron in Euclidean space", the examiner respectfully disagrees.

Viniotis states that the solution to the Linear Programming problem is an extreme point (Page 654, CL4, Para 6); extreme points form a polyhedron (Page 654, CL4, Para 6). One of ordinary skill in the art would have known that such polyhedron existed in the Euclidean space (a multi-dimensional space). The constraints of the linear program are lines in the multi-dimensional space forming the edges of the polyhedron. The constraints are defined by the states. Therefore, the state space of the linear program exists in an Euclidean space and is defined by a polyhedron. It is well known that a Markov decision Problem (MDP) is equivalent to a Linear Program; a MDP problem can be generally formulated as an equivalent Linear Program (Page 652, CL1, Para 4). Therefore, one of ordinary skill in the art would conclude that a state space for the MDP is a polyhedron in a Euclidean space.

7.2 As per the Appellant's argument that "the Office Action asserts that Viniotis teaches one or more actions that are feasible in a state of the state space are linearly constrained with respect to the state at Page 653, CL1, Para 1 and Para 2; Page 652, CL2, Para 7; however, at the indicated locations, Viniotis merely states ...it can be seen that Viniotis teaches only that a linear cost functional that involves the state is linear; however, these portions in Viniotis do

not teach or suggest that actions that are feasible in a state of the state space are linearly constrained with respect to the state in the context where a state space for the MDP is a polyhedron in a Euclidian space", the examiner respectfully disagrees.

Viniotis states that the state is a linear function of the control actions (Page 652, CL2, Para 8). One of ordinary skill in the art knows that if  $x$  is a linear function of  $y$ , then  $y$  is a linear function of  $x$ . Therefore, it is clear that actions are linear functions of state. Selecting an optimal policy (set of actions) reduces to minimizing a linear functional; this minimization is constrained, since the states generated by the policy have to belong to the state space, a subset of nonnegative integers (Page 653, CL1, Para 1). Therefore, it is obvious that the actions are constrained by the state, where the state space is in the Euclidean space.

7.3 As per the Appellant's argument that "the Office Action asserts that Viniotis teaches building a value function for the state using representations that facilitate the computation of approximately optimal actions at any given state by linear programming at Page 653, CL1, Para 9 to Page 654, CL1, Para 4 and Page 652, CL2, Para 8; ... Viniotis merely states ...it can be seen that Viniotis teaches only the formulation of an MDP and the definition of a value function; however, the indicated locations in Viniotis cannot be interpreted as teaching the limitations of the Appellant's claim directed to "building approximations from above and from below to a value function for the state using representations that facilitate the computation of approximately optimal actions at any given state by linear programming" ", the examiner takes the position that the examiner used the above section as reference only for building a value function for the state using representations and facilitating the computation of approximately optimal actions at any given state by linear programming.

7.4 As per the Appellant's argument that "the Office Action asserts that Schneider teaches building a value function for the state using representations that facilitate the computation of approximately optimal actions at any given state by linear programming at Page 2726, CL1, Para 3 and 4; ... Schneider merely states ...it can be seen that Schneider teaches only a Markov Decision Process ...", the examiner takes the position that the examiner used the above section as reference only for teaching a computerized apparatus for solving stochastic control problems of linear systems in high dimensions comprising a computer and a logic performed by a computer for modeling a structured Markov Decision Process.

7.5 As per the Appellant's argument that "the Office Action states that Schneider teaches building one or more approximations from above and from below to a value function for the state using representations at Page 2722, CL1, Para 2 and Page 2724, CL2, Para 6; ... however, the indicated sections in Schneider cannot be interpreted as teaching "building approximations from above and from below to a value function for the state using representations that facilitate the computation of approximately optimal actions at any given state by linear programming", the examiner respectfully disagrees.

Schneider teaches that the solution to the MDP is a value function and a method for generating an approximate value of this function (Page 2722, CL1, Para 2). Schneider also teaches that the solution to an MDP is an approximate value function (Page 2724, CL2, Para 6). Schneider teaches that the value function can be represented as a function of states and actions (Page 2725, CL1, Para 1). Trajectories through the MDP model are generated repeatedly using the current approximation of the value function (Page 2725, CL2, Para 4). For noisy versions, one could use noisy outcomes directly from the stochastic simulation (Page 2726, CL1, Para 3). It is inherent that when noise is introduced, the approximations to the value function will be determined by the amplitude of the noise and will thus be limited from above and from below.

Appellant's attorney disagrees. The references, taken individually or in combination, do not disclose the specific combination of elements set forth in Appellant's independent claims 1, 13 and 25.

As a general matter, the prior art simply formulates a discrete MDP in terms of linear programming, which is well known. The Appellant's invention, on the other hand, is a more general method that works in a continuous state space, continuous action setting. The Appellant's invention attempts to approximate the correct value function, with which acting optimally in each state requires solving a Linear Programming (LP) problem that incorporates this value function. The prior art does not teach or suggest these aspects of the Appellant's invention.

Turning to specifics, there are numerous examples where the references are misinterpreted by the Office Action.

For example, the Office Action asserts that Viniotis teaches "a state space for the MDP is a polyhedron in a Euclidean space," at the following locations:

Viniotis: page 654, CL2, Lemma 2

Lemma 2: If  $A$  is a totally unimodular matrix, the extreme points of the polyhedron  $\{y: Ay \leq b\}$ , where the vector  $b$  is integer-valued, are vectors with integer components.

Viniotis: Page 654, CL1, Para 6

Consider the LP problem (P), where now  $e$ ,  $A$ ,  $b$  are functions of a (vector-valued) parameter  $x \in \mathbb{R}^n$ . Sensitivity analysis studies how the optimal value function of (P) varies when the parameters of the model (i.e.,  $e$ ,  $A$ ,  $b$ ) vary as functions of  $x$ . In the queueing control problems of interest to us,  $x$  represents

the initial state of the queueing system. Moreover, only  $b$  depends on  $z$ , in a linear fashion. That is,  $b = b_0 + Fx$ , where  $b_0, F$  are (problem-dependent) constants [14].

The Office Action imputes more into Viniotis than it actually teaches. In Viniotis,  $A$  is a constraint matrix, not a state space. Nowhere does Viniotis refer to a state space for the MDP as a polyhedron in a Euclidean space.

In another example, the Office Action asserts that Viniotis teaches “one or more actions that are feasible in a state of the state space are linearly constrained with respect to the state,” at the following locations:

Viniotis: page 653, CL1, Para 1 and 2

Thus, any linear cost functional that involves the state (e.g., delay), is linear in the controls  $z_k$ . Selecting an optimal policy, therefore, reduces to minimizing a linear functional; this minimization is constrained, since the states generated by the policy have to belong to the state space  $S$ , a (possibly unbounded) subset of the nonnegative integers. From the state equation, the constraints are also linear in the control. But minimization of a linear functional over a linear constraint set is the subject of Linear Programming.

There are some points that need attention. In a Linear Program, the control variables are allowed to take values in a continuum, e.g.,  $[0,1]$  or  $\mathbb{R}^n$ . In (an unconstrained) MDP problem, the controls are integer-valued. For example, in resource allocation problems, where there are  $N+1$  distinct actions available,  $z_k \in \{0,1, \dots, N\}$ . Thus when reformulating the problem as a Linear Program, we in fact “enlarge” the solution space. This will not be a problem if existence of integer-valued optimal solutions is shown.

Viniotis: page 652, CL2, Para 7

In the next section we briefly present the technicalities of the formulation of the MDP problem as a linear program; we use the notation developed in [7]. The reader may find the missing details in [7,14].

Viniotis: Page 652, CL2, Para 8

Briefly, the procedure is as follows. From equation (1) (or (2)) the state is a linear function of the control actions  $z_k$ .

The above portions of Viniotis do not teach or suggest that actions that are feasible in a state of the state space are linearly constrained with respect to the state, in the context where a state space for the MDP is a polyhedron in a Euclidean space. Instead, the above portions of Viniotis merely state that the state is a linear function of the control actions  $z_k$ .

In another example, the Office Action asserts that Viniotis teaches "building a value function for the state using representations that facilitate the computation of approximately optimal actions at any given state by linear programming," at the following locations:

Viniotis: page 653, CL1, Para 9 to page 654, CL1, Para 4

Let  $Z$  be the set of all admissible policies; let  $Z_I$  be the subset of policies in  $Z$  that are integer-valued. Define the  $\beta$ -discounted, finite horizon, expected cost of policy  $z$ , when the system starts from state  $x$  at time  $k = 0$ , and is allowed to "move" for  $n$  steps (i.e., perform  $n$  transitions), as

(Eqn.(5))

where  $L(z_k)$  is a linear function of the state trajectory and the control process  $z$ ; it has the interpretation of an instantaneous cost. A fairly general form for  $L$ , that fits our purposes is

(Eqn.(6))

where  $c, d$  are properly dimensioned vector constants. In resource allocation problems, where delay is the cost, we have  $d = 0$ ; in pure blocking systems, we choose  $c = 0$ .

To show the exact dependence of  $J_n(z, z)$  on  $x$  and  $z$ , let us rewrite (4) as (Eqn.(7))

Then since  $x$  is constant and (Eqn.), where  $p$  denotes the probability distribution on  $\Omega^n$ , we have

(Eqn.(8))

Equation (8) stresses the fact that the cost function is linear in the variables  $z_k(w^k)$ .\* The dependence of the cost on the probability distribution, the transitions and the constants  $c, d$  is "hidden" in  $\gamma_k(w^k)$ , to emphasize the dependence of the cost on the policy  $z$ . The exact form of  $\gamma_k(w^k)$  can be routinely determined for the specific problem in hand [14]. We need only mention that  $\gamma_k(w^k)$  is independent from the control policy  $z$  and the initial state  $x$ . For the purposes of the discussion in this section, the exact form of  $\gamma_k(w^k)$  is irrelevant.

From (8) we see that the optimal policy is the one that minimizes the second term in the right hand side. From (7) the constraints fall in general into two categories:

(a) nonnegativity of states, namely

(Eqn.(9))

(b) boundedness of states, namely

(Eqn.(10))

where  $U$  is the bound. Since the constraints in (10) ( $\leq$ ) are easily converted into constraints as in (9), we shall concentrate on constraints of the form (9) only.

Summarizing, the LP equivalent problem may take the form

$$\begin{array}{ll} \min cZ & (P) \\ AZ \leq b \end{array}$$

This form is suitable to present results from sensitivity analysis.

Remark. The control variables are (Eqn.), and thus there is only a finite number of them. The constraint matrix  $A$  has elements that depend only on the transitions  $\xi_k(w^k)$ . The vector  $b$  depends only on the initial state  $z$ .

We have allowed  $z_k(w^k)$  to take values in  $[0,1]$ . For sensitivity analysis,  $x$ , the initial state of the queueing system, should be also continuously-valued. In this case, the trajectory  $i$  will be continuously-valued; such a trajectory does not of course correspond to a real queueing system.

If, however,  $x, z_k(w^k)$  are restricted to take integer-valued values only, then  $i$  will be integer-valued; in this case it does represent the evolution of the queueing system. The optimal cost function of the MDP in this case is given by\* (Eqn.(11))

This is actually a problem in Integer Programming, the sensitivity analysis of which is not as well developed as that of a Linear Program. If we remove the restriction on integer-valued policies (and states), we have the above mentioned Linear Programming problem (P). Let

(Eqn.(12))

denote the optimal value function of problem (P). It is  $W_n(z)$  for which results from sensitivity analysis apply. We wish to emphasize here that the functions  $W_n, V_n$  are quite different; first of all, they are even defined on different domains. If we can make, however, a suitable connection between them, then we can relate the properties of  $W_n$  (which we shall determine) to those of  $V_n$  (which we want).

Such a connection is indeed possible, if the Linear Program in (12) admits an integer-valued solution. In this case, for integer-valued  $x$ , (11) and (12) refer to the same problem. The optimal value function of the LP "contains" in some sense the optimal value of the MDP: we can recover  $V_n(x)$  by "interpolating"  $W_n(x)$  at the integer-valued points of its domain. Consequently, all the properties of  $W_n(x)$  are automatically properties of  $V_n(x)$  as well.

Viniotis: page 652, CL2, Para 8

Briefly, the procedure is as follows. From equation (1) (or (2)) the state is a linear function of the control actions  $z_k$ .

The above portions of Viniotis do not teach that building a value function for the state using representations and facilitating the computation of approximately optimal actions at any given state by linear programming. Instead, the above portions of Viniotis teach only the formulation of an MDP and the definition of a value function, as well as that the state is a linear function of the control actions.

In another example, the Office Action asserts that Schneider teaches "building one or more approximations from above and from below to a value function for the state using

representations that facilitate the computation of approximately optimal actions at any given state by linear programming,” at the following locations:

Schneider: page 2726, CL1, Para 3 and 4

Our experiments consider both deterministic and noisy versions of the problem. To build the deterministic version of the problem, we ran long (stochastic) simulations for each of the 421 actions and cached the mean observed production rate for each. For the noisy versions, we could have used noisy outcomes directly from the stochastic simulation, but instead we simply added Gaussian noise to the cached, deterministic production rates. This enabled our experiments to run significantly faster, and also allowed us to easily generate empirical results with varying amounts of noise.

Table 1 shows experimental results. The computation times reported are on a 200 MHz Pentium Pro. The first section contains results for the case where the factory output is deterministic and known. The purpose of the first two lines is to delimit the range of results we should expect from good algorithms. The “Random” algorithm builds a schedule by choosing 8 configurations at random, and it loses an enormous amount of money. Much of the cost is due to heuristic penalties for failing to satisfy customer demand.

Schneider: page 2722, CL1, Para 2

In this paper, we describe a Markov Decision Process (MDP) formulation of production scheduling which captures stochasticity, while retaining the ability to construct a schedule to meet demand forecasts. The solution to this MDP is a value function, specific to the current demand forecasts, which can be used to generate optimal scheduling decisions online. We then describe an industrial application and a reinforcement learning method for generating an approximate value function in this domain. Our results demonstrate that in both deterministic and noisy scenarios, value function approximation is an effective technique.

Schneider: page 2724, CL2, Para 6

Here we describe a principled approach to generating closed-loop production scheduling policies with reinforcement learning methods. It combines the capabilities of both optimal control and AI search based methods. The approach is based on representing the problem as an MDP and representing the solution as an approximate value function. In contrast to many optimal control based methods, it produces a time-dependent policy specifically built to match current demand forecasts, rather than a time-invariant policy that ignores all demand information other than the current rate. Our experiments also demonstrate the ability to search hundreds of alternative factory configurations.



Schneider: Page 2725, CL1, Para 1

Abstractly, a Markov Decision Process (MDP) is defined by a state space  $X$ , action set  $A$ , immediate reward function  $R(x, a)$ , and probabilistic transition model  $P(x'|x, a)$ . The solution to the MDP is a policy  $\pi^*: X \rightarrow A$  which, if followed by the agent, will maximize the expected long-term sum of rewards attainable starting from any state  $x$ . Dynamic programming methods tabulate this optimal cumulative reward in the optimal value function  $V^*(x)$ , which is the unique solution to the Bellman equations [3]:

(Eqn. 1)

Once  $V^*$  is computed, the optimal policy  $\pi^*$  is immediately obtained by choosing any action which instantiates the max in Eq. 1.

Schneider: Page 2725, CL2, Para 4

- The action set consists of all legal factory configurations. We assume a discrete-time model, so the configuration chosen at time  $t$  will run unchanged until time  $t + 1$ .

The above portions of Schneider do not teach or suggest building one or more approximations from above and from below to a value function for the state using representations that facilitate the computation of approximately optimal actions at any given state by linear programming. Instead, the above portions of Schneider teach only a Markov Decision Process (MDP) formulation of production scheduling which captures stochasticity. Further, the above portions of Schneider merely describe how a Markov Decision Process (MDP) is defined by a state space  $X$ , action set  $A$ , immediate reward function  $R(x, a)$ , and probabilistic transition model  $P(x'|x, a)$ . Finally, the above portions of Schneider merely describe how the solution to the MDP is an approximate value function, specific to the current demand forecasts, which can be used to generate optimal scheduling decisions online.

Dangat and Hedlund fail to overcome these deficiencies in the combination of Viniotis and Schneider. Recall that Dangat and Hedlund were cited only against the dependent claims.

The various elements of Appellant's claimed invention together provide operational advantages over Viniotis, Schneider, Dangat, and Hedlund. In addition, Appellant's invention solves problems not recognized by Viniotis, Schneider, Dangat, or Hedlund.

Thus, Appellant submits that independent claims 1, 13, and 25 are allowable over Viniotis, Schneider, Dangat, and Hedlund. Appellant's dependent claims 2-12, 14-24, and 26-36 are submitted to be allowable over Viniotis, Schneider, Dangat, and Hedlund in the same

manner, because they are dependent on independent claims 1, 13, and 25, respectively, and thus contain all the limitations of the independent claims. In addition, dependent claims 2-12, 14-24, and 26-36 recite additional novel elements not shown by Viniotis, Schneider, Dangat, or Hedlund.

2. Claims 3, 15 and 27

With regard to claims 3, 15 and 27, which recite that “the action space and the state space are continuous and related to each other through a system of linear constraints,” the Office Action asserts that Viniotis teaches these limitations at Page 652, CL2, Para 7 and Page 653, CL1, Para 1. However, at the indicated locations, Viniotis merely describes the formulation of the MDP problem as a linear program, and merely describes the constraints as being linear.

3. Claims 4, 16 and 28

With regard to claims 4, 16 and 28, which recite that “the value function is convex and the method further comprises efficiently learning the value function in advance and representing the value function in a way that allows for real-time choice of actions based thereon,” the Office Action asserts that Viniotis teaches that the value function is convex, at Page 652, CL1, Para 4 and that Schneider teaches that the logic further comprises efficiently learning the value function in advance and representing the value function in a way that allows for real-time choice of actions based thereon, at Page 2722, CL1, Para 2, as that allows producing a time-dependent action policy specifically built to match the current demand forecasts and other system states, at Page 2724, CL2, Para 4. However, at the indicated locations, Viniotis merely describes the optimal cost function of a tandem queueing system as being convex, and Schneider merely states that the solution to an MDP is a value function, and that the value function may be an approximate value function.

4. Claims 5, 17 and 29

With regard to claims 5, 17 and 29, which recite that “the linear function is approximated both from above and from below by piecewise linear and convex functions,” the Office Action

asserts that Viniotis teaches that the linear function is represented by piecewise linear and convex functions, at Page 652, CL1, Para 4 and Page 654, CL1, Lemma 1, and that Schneider teaches the linear function is approximated both from above and from below, at Page 2722, CL1, Para 2, Page 2724, CL2, Para 6, and Page 2722, CL2, Para 2. However, at the indicated locations, Viniotis merely describes the optimal cost function of a tandem queueing system as being convex, that the optimal value function may be piecewise linear, continuous and convex, and Schneider merely states that solutions to large-scale MDPs may be by value function approximation.

5. Claims 6, 18 and 30

With regard to claims 6, 18 and 30, which recite that “the domains of linearity of the piecewise linear and convex functions are not stored explicitly, but rather are encoded through a linear programming formulation,” the Office Action asserts that Viniotis teaches that the domains of linearity of the piecewise linear and convex functions are not stored explicitly, but rather are encoded through a linear programming formulation, at Page 652, CL1, Para 4, Page 654, CL1, Lemma 1, and Page 653, CL1, Para 9 to Page 654, CL1, Para 4). However, at the indicated locations, Viniotis merely describes the optimal cost function of a tandem queueing system as being convex, that the optimal value function is piecewise linear, continuous and convex, and that the state is a linear function of the control actions.

6. Claims 7, 19 and 31

With regard to claims 7, 19 and 31, which recite that “the domains of linearity of the piecewise linear and convex functions allow the functions to be optimized and updated in real-time,” the Office Action asserts that Viniotis teaches that the domains of linearity of the piecewise linear and convex functions allow the functions to be optimized and updated, at Page 652, CL1, Para 4, Page 654, CL1, Lemma 1, and Page 654, CL1, Para 6 to Para 8, while Schneider teaches that the domains of linearity of the functions allow the functions to be optimized and updated in real-time, at Page 2722, CL1, Para 2, Page 2724, CL2, Para 6, as that allows producing a time-dependent action policy specifically built to match the current demand

forecasts and other system states, at Page 2724, CL2, Para 4. However, at the indicated locations, Viniotis merely describes the optimal cost function of a tandem queueing system as being convex, and the optimal value function of problem is piecewise linear, continuous and convex.

7. Claims 8, 20 and 32

With regard to claims 8, 20 and 32, which recite that “the value function can be efficiently approximated both from above and from below,” the Office Action asserts that Schneider teaches that the value function can be efficiently approximated both from above and from below, at Page 2722, CL1, Para 2 and Page 2724, CL2, Para 6, that value function approximation is an effective technique for both deterministic and noisy scenarios, at Page 2722, CL1, Para 2, and that approximation allows solving large scale MDPs, at Page 2722, CL2, Para 2. However, at the indicated locations, Schneider merely describes the value function approximation for solving MDPs.

8. Claims 9, 21 and 33

With regard to claims 9, 21 and 33, which recite that “the approximations can be repeatedly refined,” the Office Action asserts that Schneider teaches that the approximations can be repeatedly refined, at Page 2725, CL2, Para 4, as that allows producing a time-dependent action policy specifically built to match the current demand forecasts and other system states, at Page 2724, CL2, Para 4. However, at the indicated locations, Schneider merely describes representing a problem as an MDP and representing the solution as an approximate value function.

9. Claims 10, 22 and 34

With regard to claims 10, 22 and 34, which recite that “the value function can be efficiently approximated from above based on knowledge of upper bounds on the function at each member of a selected set of states,” the Office Action asserts that Schneider teaches that the value function can be efficiently approximated from above based on knowledge of upper bounds

on the function at each member of a selected set of states, at Page 2722, CL1, Para 2, Page 2724, CL2, Para 6, and Page 2725, CL2, Para 3, as value function approximation is an effective technique for both deterministic and noisy scenarios, at Page 2722, CL1, Para 2, and approximation allows solving large scale MDPs, at Page 2722, CL2, Para 2. However, at the indicated locations, Schneider merely describes generating an approximate value function as a solution for an MDP, for example, using global or local polynomial regression.

10. Claims 12, 24 and 36

With regard to claims 12, 24 and 36, which recite that “the value function can be approximated successively,” the Office Action asserts that Schneider teaches that the value function can be approximated successively, at Page 2725, CL2, Para 4, as that allows producing a time-dependent action policy specifically built to match the current demand forecasts and other system states, at Page 2724, CL2, Para 4. However, at the indicated locations, Schneider merely describes an action set consisting of all legal factory configurations, and representing the problem as a MDP and representing the solution as an approximate value function.

H. Arguments Directed To The First Grounds for Rejection: Whether claims 2, 14, and 26 are obvious under 35 U.S.C. §103(a) over Viniotis in view of Schneider and further in view of Dangat et al., U.S. Patent No. 5,971,585 (Dangat).

1. Claims 2, 14 and 26

With regard to claims 2, 14 and 26, which recite that “the MDP comprises a supply chain planning process,” the Office Action asserts that Dangat teaches that MDP comprises a supply chain planning process, at the Abstract, CL1, L7-21, and CL6, L5-9. However, at the indicated locations, Dangat merely describes supply chain analysis, but not with the use of MDPs.

I. Arguments Directed To The Third Grounds for Rejection: Whether claims 11, 23, and 35 are obvious under 35 U.S.C. §103(a) over Viniotis in view of Schneider and further in view of Hedlund et al., "Optimal control of hybrid systems," IEEE, 1999 (Hedlund).

1. Claims 11, 23 and 35,"

With regard to claims 11, 23 and 35, which recite that "the value function can be efficiently approximated from below based on linear functions that lie below the convex value function," the Office Action asserts that Hedlund teaches that the value function can be efficiently approximated from below based on linear functions that lie below the convex value function, at Page 3972, CL1, Para 1, Page 3973, CL1, Para 3, and Page 3977, CL1, Para 1, as that would provide a lower bound on the optimal cost in terms of linear programming, at Page 3972, CL2, Para 2. However, at the indicated locations, Hedlund merely describes a value function that preserves a lower bound property, a set of value functions, and a lower bound on an optimal cost function.

VIII. CONCLUSION

In light of the above arguments, Appellant's attorney respectfully submits that the cited references do not anticipate nor render obvious the claimed invention. More specifically, Appellant's claims recite novel physical features, which patentably distinguish over any and all references under 35 U.S.C. §§ 102 and 103.

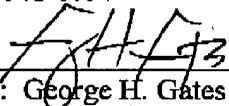
As a result, a decision by the Board of Patent Appeals and Interferences reversing the Examiner and directing allowance of the pending claims in the subject application is respectfully solicited.

Respectfully submitted,

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## APPENDIX

1. A method for solving, in a computer, stochastic control problems of linear systems in high dimensions, comprising:

(a) modeling, in the computer, a structured Markov Decision Process (MDP), wherein a state space for the MDP is a polyhedron in a Euclidean space and one or more actions that are feasible in a state of the state space are linearly constrained with respect to the state; and

(b) building, in the computer, one or more approximations from above and from below to a value function for the state using representations that facilitate the computation of approximately optimal actions at any given state by linear programming.

2. The method of claim 1, wherein the MDP comprises a supply chain planning process.

3. The method of claim 1, wherein the action space and the state space are continuous and related to each other through a system of linear constraints.

4. The method of claim 1, wherein the value function is convex and the method further comprises efficiently learning the value function in advance and representing the value function in a way that allows for real-time choice of actions based thereon.

5. The method of claim 1, wherein the linear function is approximated both from above and from below by piecewise linear and convex functions.

6. The method of claim 5, wherein the domains of linearity of the piecewise linear and convex functions are not stored explicitly, but rather are encoded through a linear programming formulation.

7. The method of claim 5, wherein the domains of linearity of the piecewise linear and convex functions allow the functions to be optimized and updated in real-time.



8. The method of claim 1, wherein the value function can be efficiently approximated both from above and from below.
9. The method of claim 1, wherein the approximations can be repeatedly refined.
10. The method of claim 1, wherein the value function can be efficiently approximated from above based on knowledge of upper bounds on the function at each member of a selected set of states.
11. The method of claim 1, wherein the value function can be efficiently approximated from below based on linear functions that lie below the convex value function.
12. The method of claim 1, wherein the value function can be approximated successively.
13. A computerized apparatus for solving stochastic control problems of linear systems in high dimensions, comprising:
  - (a) a computer;
  - (b) logic, performed by the computer, for modeling a structured Markov Decision Process (MDP), wherein a state space for the MDP is a polyhedron in a Euclidean space and one or more actions that are feasible in a state of the state space are linearly constrained with respect to the state; and
  - (c) logic, performed by the computer, for building one or more approximations from above and from below to a value function for the state using representations that facilitate the computation of approximately optimal actions at any given state by linear programming.
14. The apparatus of claim 13, wherein the MDP comprises a supply chain planning process.
15. The apparatus of claim 13, wherein the action space and the state space are continuous and related to each other through a system of linear constraints.

16. The apparatus of claim 13, wherein the value function is convex and the logic further comprises efficiently learning the value function in advance and representing the value function in a way that allows for real-time choice of actions based thereon.

17. The apparatus of claim 13, wherein the linear function is approximated both from above and from below by piecewise linear and convex functions.

18. The apparatus of claim 17, wherein the domains of linearity of the piecewise linear and convex functions are not stored explicitly, but rather are encoded through a linear programming formulation.

19. The apparatus of claim 17, wherein the domains of linearity of the piecewise linear and convex functions allow the functions to be optimized and updated in real-time.

20. The apparatus of claim 13, wherein the value function can be efficiently approximated both from above and from below.

21. The apparatus of claim 13, wherein the approximations can be repeatedly refined.

22. The apparatus of claim 13, wherein the value function can be efficiently approximated from above based on knowledge of upper bounds on the function at each member of a selected set of states.

23. The apparatus of claim 13, wherein the value function can be efficiently approximated from below based on linear functions that lie below the convex value function.

24. The apparatus of claim 13, wherein the value function can be approximated successively.

25. An article of manufacture embodying logic for solving stochastic control problems of linear systems in high dimensions, the logic comprising:

(a) modeling a structured Markov Decision Process (MDP), wherein a state space for the MDP is a polyhedron in a Euclidean space and one or more actions that are feasible in a state of the state space are linearly constrained with respect to the state; and

(b) building one or more approximations from above and from below to a value function for the state using representations that facilitate the computation of approximately optimal actions at any given state by linear programming.

26. The article of manufacture of claim 25, wherein the MDP comprises a supply chain planning process.

27. The article of manufacture of claim 25, wherein the action space and the state space are continuous and related to each other through a system of linear constraints.

28. The article of manufacture of claim 25, wherein the value function is convex and the logic further comprises efficiently learning the value function in advance and representing the value function in a way that allows for real-time choice of actions based thereon.

29. The article of manufacture of claim 25, wherein the linear function is approximated both from above and from below by piecewise linear and convex functions.

30. The article of manufacture of claim 29, wherein the domains of linearity of the piecewise linear and convex functions are not stored explicitly, but rather are encoded through a linear programming formulation.

31. The article of manufacture of claim 29, wherein the domains of linearity of the piecewise linear and convex functions allow the functions to be optimized and updated in real-time.

32. The article of manufacture of claim 25, wherein the value function can be efficiently approximated both from above and from below.

33. The article of manufacture of claim 25, wherein the approximations can be repeatedly refined.

34. The article of manufacture of claim 25, wherein the value function can be efficiently approximated from above based on knowledge of upper bounds on the function at each member of a selected set of states.

35. The article of manufacture of claim 25, wherein the value function can be efficiently approximated from below based on linear functions that lie below the convex value function.

36. The article of manufacture of claim 25, wherein the value function can be approximated successively.